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THE GOLDEN SECTION AS A SACRED SYMBOL

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Abstract: *The Fibonacci Spiral or the Golden Spiral represents one of the sacred geometrical symbols. Being a logarithmic sequence, it generated the sacred proportion that established the geometrical relationships used to design the ancient Greek temples. The intervals among the columns or those among different parts of the temple facade correspond to the same intervals used in music or in poetry. In Pre-Renaissance and Renaissance time, the sacred mathematical conception, given by Pythagoras and developed by Plato, was considered the ideal model by Leonardo, Piero de la Francesca and by Luca Pacioli. In modern times the same Golden Section is to be found in Debussy's and Bartók's compositions or in Eisenstein's films. Starting as a mathematical sequence that stands for the aesthetic unity between form and beauty, Sectio Aurea could be described as attributing a religious value to the works of art belonging to architecture, painting, poetry, music, furniture design, Stradivari's violins conception and even cinematography. The aim of this paper is to analyze the connection the Golden Ratio realizes among these artistic fields on aesthetic grounds.*

Before starting, it is important to define the term golden section also known as the golden number, the golden ratio and the golden spiral. One of the first definitions for the golden section as the derived proportion resulted from the simple division of a segment into media and extreme ratio was given around 300 BC by Euclid from Alexandria.

Fig. 1

Considering that the segment AB is evidently bigger than the segment AC then the segment AC must be bigger than the segment CB. This could be translated as follows: if the proportion between the length of the segment AC and of the segment CB is identical with the proportion between the length of the segment AB and of the segment AC, that means the whole segment AB was divided into extreme and mean ratio or in a Golden Section. According to the mathematical demonstrations the exact value of the Golden Section (the proportion between the segment AC and the segment CB, as shown in the *Fig. 1*) is the famous number that never ends and that never repeats itself: 1, 6180339887... . The symbol for the Golden Section is the Greek letter τ (tau), taken from

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the Greek word 'τομή' which means *cut* or *section.* But at the beginning of the 20^{th} century, the American mathematician Mark Barr used the letter **ϕ** (phi)[1] to indicate the Golden Section. There were many scholars, such as Euclid and Pythagoras in ancient Greece, or the Italian mathematician Leonardo from Pisa, in medieval times, or the physicist Johannes Kepler in Renaissance, or the contemporary physicist Oxford Roger Penrose, who dedicated their researches to this particular proportion and its specific properties. The fascination of the Golden Proportion could not be limited to the field of mathematics or physics, but it was extended to other fields, among which biology, music, history, architecture, psychology, literature and art.

The term Golden Section was first mentioned in the second edition of Martin Ohm's[2] book *Die Reine Elementar-Mathematik* from 1835. The fact that the term Goldener Schnitt was not used in the first edition of the same book published in 1826 shows that its popularity rose after 1830. Indubitably, after the release of Ohm's book, the term Golden Section was frequently used in the German history of mathematics and in art history. The English debut of the term was considered the article[3] on aesthetics written by James Sully which appeared in the ninth edition of Encyclopædia Britannica from 1873. The following article intends to analyse the connection between the Golden Section and several artistic representations.

The Division in Extreme and Mean Ratio

One of the first definitions of the Golden Section was given by Euclid in his book *Elements*, but there must be mentioned the link between the Golden Ratio and Plato. There are two things that are considered the basis of this link: the incommensurability and the Platonic solids (Kraut 177-195). The Golden section is represented by a number which is neither a whole number nor a fraction. This means that the proportion between the two lengths AC and CB (Fig. 1) could not be expressed using a mathematical fraction.[4] Two established lengths that have not such a common measure are called incommensurable. This transforms the Golden Section in an irrational number. In *Hippias Major*, Plato noticed that an even number could be the sum of two odd numbers and in the same way the sum of two irrational numbers could be either rational or irrational. As it was mentioned before, **ϕ** is an irrational number and this sustains the fact that a rational segment, having the length as its unity, divided through a Golden Section could be the

[1] The Greek letter ϕ derives from Phidias, the name of a great Greek sculptor who lived approximately between 490 and 430 before Christ. The American mathematician intended to honour the sculptor's name mainly because most of the art historians admitted that Phidias used frequently the Golden Section in his sculptures.

[2] The German mathematician Martin Ohm was the brother of the famous German physicist Georg Simon Ohm.

[3] In his article, Sully referred to the experimental research initiated by the German psychologist Gustav Theodor Fechner regarding the superiority of the Golden Section as a visible proportion in any work of art.

[4] In other words, it is impossible to find a common measure which could be included let's say by thirty one times in AC and by nineteen times in CB.

sum of two irrational segments and this represents the incommensurability of the segment. As related to the Platonic solids, it could be said that Plato tried to explain in his dialogue *Timaios* the structure of the matter using the regular polyhedrons.[5] The five Platonic solids have the following properties: they are the only polyhedrons whose faces are identical and equilateral and each polyhedron enters a sphere, having all its vertices placed on the sphere (Fig. 2). The Platonic solids are tetrahedron (three triangles meet at each vertex), cube (three squares meet at each vertex), octahedron (four triangles meet at each vertex), dodecahedron (three pentagons meet at each vertex) and icosahedrons (five triangles meet at each vertex).

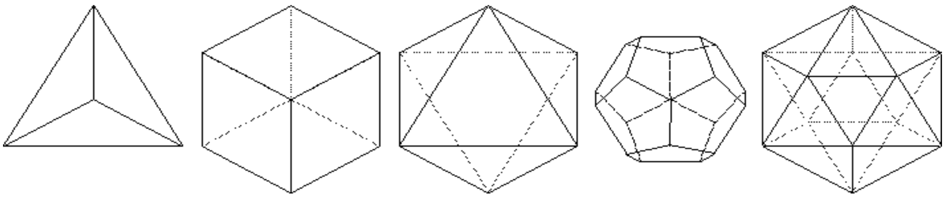

Fig. 2

In his dialogue, Plato combined Empedocles' theory referring to the four fundamental elements of the matter (ground, water, air and fire) with Democritus' theory concerning the existence of the indivisible particles. Thus, Plato's theory affirmed that each of the four fundamental elements corresponds to a different type of fundamental particle and is represented by a Platonic solid (Jowett 31-35). But Plato's theory was more than a symbolic association. For Plato what really counts are not the complex phenomena concerning the universe, but the underlying symmetries which never change.

The same idea could be found in the Euclid's Elements. The interest for the proportion and symmetries governs few of the Books that belong to Elements. The definition of the Golden Section, as extreme and mean ratio, was given for the first time in the Book II. The second definition that examined the Golden Section[6] as the divine and perfect proportion appeared in the Book VI. Here it was demonstrated that in a regular pentagon, the proportion between the diagonal and the side equals ϕ. This fact illustrated the possibility of dividing a segment into a Golden Section and also of constructing a regular pentagon (Heath 88-90). Considering Euclid's definition, in the following centuries, the Greek mathematicians got new results[7] that implicated the Golden Section. Later on Ptolemy (Claudius Ptolemaus) created one of the first equivalents of the trigonometric tables for several angles. One of the last mathematicians that had a great interest in geometry and also in the Golden Section was Pappus from Alexandria. In his Synagoge, written around 340 AD, Pappus proposed a new method of constructing the

[5] This theory was used before by Pythagoras and by Theaitetos.
[6] Euclid was mainly interested in the geometrical analysis and interpretation of the Golden Section and also in the possibilities of using it as means of constructing the pentagon and some Platonic solids.
[7] The *Supplement* of Euclid's *Elements* contains an important theorem which refers to the possibility of including the dodecahedron and the icosahedrons in the same sphere.

dodecahedron or the icosahedrons. He also introduced new methods of geometrical comparison of the volumes of the Platonic solids, all involving the Golden Section. Pappus' comments to Euclid's theory concerning the irrational numbers made an interesting presentation[8] of the evolvement of the irrational numbers. Unfortunately after Pappus' death the decline of mathematics in general and of geometry in particular could not be stopped and the interest for the Golden Section was considerably diminished. Once the ascension of the Islamic empire began, the Muslim world became an important centre for the mathematical studies.[9] Most of the Arabic mathematical studies were very algebraic and referred to the Golden Section only tangentially.

In the history of the Golden Section a very important role was imposed by Fibonacci who extended the studies upon the applications of this logarithmic sequence. His direct contributions could be found in *Practica Geometriae* written in 1223. Here new methods of calculation for the pentagon diagonal and area, together with the methods of obtaining the mathematical values of the pentagon and decagon sides starting from the diameter of the inscribed or circumscribed circle and from the dodecahedron and icosahedrons volumes were presented in a strict relation to the Golden Section. The main important contribution to the analysis of the Golden Section was given by Fibonacci in his book *Liber Abaci*, written in 1202. According to his writings, Fibonacci' numbers and sequence represent the numbers in the following integer sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 and so on. These numbers are closely connected with the Golden Ratio, for example the closest rational approximations to the ratio are 2/1, 3/2, 5/3, 8/5 and so on (Dunlap 7-12). Using these numbers in geometry, the process of geometrical division will create a tiling with squares whose side lengths are successive Fibonacci numbers. By drawing circular arcs in order to connect the opposite corners of squares in the Fibonacci tiling, there could be obtained a Golden Spiral (Fig. 3).

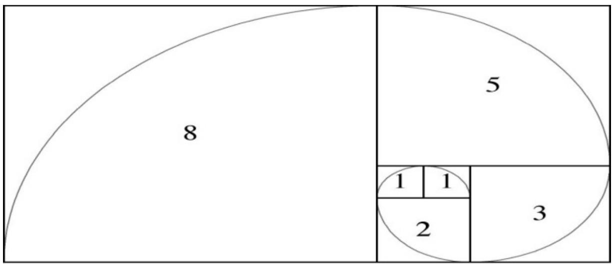

Fig. 3

The Golden Section continued to inspire many other mathematicians who gave their important contributions to the history of this fascinating proportion. Being transposed into the field of arts, the Golden Section will impress both the creators and the analysts during many centuries.

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[8] This presentation outlasted the Arabic translations.

[9] Only because of the Islamic intellectual development from the 8th century most of the Ancient mathematical studies were not lost completely. The caliph from Bagdad, al-Mamun (786-833), founded the School of Wisdom which functioned in the same way as the Alexandrian University *Museum*.

The Golden Section as Divine Proportion

Renaissance is the period when the history of the Golden Section changes its direction from mathematics to arts[10] and natural phenomena. Although the Golden Section can be found in many visual works of art created before Renaissance time, now it is the moment when the importance of mathematics as a basis of the visual perspective is recognised. Discovering the Golden Section in so many artistic representations may prove the fact that some artists used it consciously[11] but, at the same time, other artists unconsciously made use of it.

Three of the Renaissance main representatives, Pierro della Francesca, Leonardo da Vinci and Albrecht Dürer, contributed to the development of mathematics in general and, at the same, they played an important role in the researches related to the Golden Section. Three of Pierro della Francesca's books on mathematics[12] survived: De Perspectiva Pingendi, Libellus de Quinque Corporibus Regularibus and Trattato d'Abaco. Written around 1470, De Perspectiva Pingendi contains several references to Euclid's Elements. In this book, the author described the technique of creating the visual perspective based on scientific grounds. In his paintings, the perspective that was used gave a spatial frame related to all geometrical features of the figures included. For the painter, even the act of painting was considered the act of representing the geometrical figures of increasing or decreasing dimensions. Because of its importance and utility, the book became a training manual for young artists and some parts of it were included in many other books on perspective. It was the case of the algebraic results which were incorporated in Luca Pacioli's book *Summa de Arithmetica, Geometria, Proportione et Proportionalita*. This was not the only case. Most of Pierro della Francesca's works on Platonic solids, which first were written in Latin, were translated into Italian by Luca Pacioli[13] and included in his book *Divina Proporzione*. While being in Milan, Luca Pacioli finished his three volume book which was published in Venice in 1509. The first volume, *Compedio di Divina Proporzione*, contains a detailed summary of the Golden Section properties. The author thoroughly analysed the thirteen effects of the divine proportion and labelled each one of them with adjectives such as essential, singular, wonderful, supreme etc. His intention was to reveal the secret of the geometrical figures using the Golden Section.[14] In doing so, he

[10] During Antiquity, the Golden Section could be identified in the architectural plans of important buildings such as the Parthenon and the Great Pyramid.
[11] According to David Hockney, starting with 1430 there were artists who secretly used specific devices such as lenses or concave mirrors in order to create more and more realistic pictures (Hockney 246-253).
[12] In most of his books, Pierro della Francesca included the so-called *useless* problems such as the calculation of the sides of an octahedron which was inserted in a cube or the calculation of the diameter of the five small circles inserted in another circle with a known diameter. The solution of the second problem refers to the pentagon and thus to the Golden Section.
[13] Luca Pacioli was one of Pierro della Francesca's students. Distinct from other students, Pacioli was also a good mathematician.
[14] Unfortunately, the text of the book remained behind the great purpose, containing conventional mathematical formulas explained using philosophical definitions.

also used Leonardo's drawings of the solids in a skeletal and solid form representation. Probably, these were the first illustrations of the skeletal form of the solids which allow the visual distinction between the front and the back of the solid. The second volume is a detailed description of the divine proportion and its applications on architecture and on human body structure. Pacioli's conclusion specified that the human body could be divided in lots of proportions established by God and hidden in all the mysteries that are found in nature. But Pacioli did not insist on the fact that the Golden Section determines the proportion[15] for all the works of art. On the contrary, when referring to the divine proportion applied on the techniques of drawing, the author explicitly pleaded for the Vitruvian system based on rational and simple relations. The third volume is a short presentation of the five regular solids translated from Pierro della Francesca's book, *Libellus de Quinque Corporibus Regularibus.*Going further into the history of the Golden Section, there could not be omitted Dürer's important contributions which belong to his artistic and to his scientific works. *Unterweisung der Messung mit dem Zirkel und Richtscheit*, published in 1525, represented one of the first books on mathematics written in German. Here, Dürer described the method of constructing different types of curves, including the logarithmic spiral, which is related to the Golden Section, of several polygons, including the pentagon, of Platonic solids and different other solids created by the artist himself. The book was not meant to be a manual on geometry that is why the author gave only one example for his demonstrations. Most of the times, he started with a practical application which helped him to discover the fundamental theoretical aspects. In this book, the author presented the first network[16] of the polyhedrons. The fascination of the Golden Section can be noticed in Dürer's work of art *Melancholia* which contains many symbolic objects among which the Magic Square, which is considered to represent mathematics. In this square each line, column, diagonal together with the four numbers placed in the centre and the numbers placed in the four corners have the sum of 34, a Fibonacci number. In the same allegory, the two numbers placed on the bottom line give the number 1514, the date when the gravure was made. It is not possible to affirm that the German artist created this piece of art considering the Golden Section but its influence can be suggested by the symbols that were used.

Except the works of Pacioli, Leonardo and Dürer, the 16th century was poor in developing other important contributions for the history of the Golden Section. Even so, these works mentioned before were followed by a great interest in Platonism and in Pythagoras's theories. The concept of the Golden Section was now considered a bridge between mathematics and cosmology, between science and arts.

[15] Considering the Golden Section as Pacioli's canon for proportion proved to be wrong. It came from a false assertion which was made by the French mathematicians Jean Etienne Montucla and by Jérôme de Lalande in *The History of Mathematics* in the 1799 edition.
[16] These are plane bands on which the sides of the polyhedrons are drown in such a way that the figures could be depicted as individual pieces and arranged in a certain way in order to construct the tridimensional solids.

Identifying the Golden Section in Painting

Most of the analyses regarding the Golden Section in painting are related to the supposed aesthetic properties of the Golden Rectangle. One of the first examples could be *Madona Ognissanti*, one of the biggest creations of the Italian painter and architect Giotto di Bondone. Painted between 1306 and 1310, this work of art portrays the Virgin sitting and being surrounded by angels and saints arranged in a hierarchy seconded by visual perspective. The whole painting and the central characters could enter exactly the Golden Rectangle (D'Arcais 227-235). There are other examples such as *Madona Rucellai*, the work of Duccio di Bouninsegna, painted in 1285, and *Madona*, the work of Cenni di Pepo, also known as Cimabue. Considering Charles Bouleau's opinions presented in his book *The Painter's Secret Geometry*, from 1963, Pacioli's important work represented rather the end of an era than its beginnings, revealing the manner of thinking for many centuries of oral tradition (Bouleau 125-140). During this time, the Golden Section was seen as the expression of divine beauty; but the history of this perfect proportion did not confirm the fact that it was not idolized by the artists in the centuries that preceded the launch of Pacioli's book. Analyzing all the three paintings, the results may indicate that the Golden Section[17] was not used on purpose by the artists, even if it was easily recognized. Leonardo's paintings could be another example. The works of art such as *Madonna delle Rocce* or *Mona Lisa* or *Saint Jerome in the Wilderness* are only few cases in which the Golden Rectangle frames the main character; but, even if it is possible, there are no evidences that the artist visualized the paintings in relation to the Golden Section.

Besides the painters already mentioned, the same Golden Ratio can be identified in other paintings belonging to the 19th century artists. Georges Seurat's *The Circus* is a composition based on some principles given by David Sutter and adopted by the author. In order to support the idea of the golden proportion identified in the painting, Sutter admitted that when the dominant line is a horizontal line then the succession of the vertical objects are placed on that line because this series will meet the horizontal line (Hertz-Fischler 109-112).

Later, the Golden Section was used by some of the Postimpressionism representatives: Paul Sérusier,[18] Pierre Bonnard, Édouard Vuillard and Maurice Denis who founded the group *Les Nabis*. Being against the expressive way of using the form and the colour, Picasso and Georges Braque deliberately rejected any theme related to emotion. In their manner of painting, the objects, the musical instruments or the human faces were divided into very small geometrical figures and then combined in mobile perspectives. This analysis of the solids having the purpose of revealing their structures was compatible with the most of the geometrical concepts, including the Golden Section.[19]

[17] All the three paintings were painted with two centuries before the launch of Pacioli's *Divina Proporzione*.
[18] Although Sérusier's interest for the Golden Section was mainly theoretical, he used the divine proportion in order to emend the composition.
[19] Jacques Villon together with his brothers, Marcel and Raymond Duchamp, and Albert Gleizes and Francis Picabia organized in Paris, in 1912, an exhibition entitled *Section d'Or*.

Even later the Golden Section inspired a lot of artists. This is the example of an Italian painter, Mario Merz who transformed the Fibonacci numbers into an important ingredient of his works. Together with other artists such as Michelangelo Pistoletto, Luciano Fabro and Jannis Kounellis, Merz founded *Arte Povera*. Their intention was to use simple objects or materials as a protest against the dehumanized society. Merz started to use the Fibonacci numbers in his conceptual works which included these numbers in a series of spirals. In the painting entitled *Onda d'urto* from 1987, the author used newspapers with the Fibonacci numbers shinning the blue neon light. From this series of examples it could not be omitted the Dutch painter Piet Mondrian whose geometric and non-figurative style characterises a huge part of his work. Some of his compositions contain only vertical and horizontal lines, rectangles and squares. A good example is the painting entitled *Broadway Boogie-Woogie* in which all the horizontal and vertical lines are all in a geometrical relation based on the Golden Ratio. In this case, the author used only primary colours on a white background while the curves and the three-dimensional representation were completely eliminated. The examples could continue because the Golden Section is the division that never ends and since Antiquity, when it was used to design the ancient Greek temples, till today it still inspires a lot of artists.

Conclusions

Most of the attempts of justifying the falsity or the truth of the Golden Section started with the supposition that a canon of the ideal beauty in art exists and it could be used as such. The divine proportion as a perfect measure does not imply the idea that just the Golden Ratio is enough for working out an aesthetic canon. The history of art showed that the artists who created important works of art were those who ignored most of the academic principles and proposed new visions upon art. For each important artistic current, Renaissance, Impressionism, Postimpressionism, Cubism or Neo-Plasticism, the main representatives gave new artistic perspectives, suggesting new approaches to art. The Golden Section could be identified in so many artistic works, and not only in paintings, but also in music and literature, in architecture and furniture design. Being used so many times proves that it is fascinating for a lot of people: mathematicians, scientists, artists and even critics, who are convinced that it is a perfect proportion, so important for mathematics, but, unfortunately, not enough to develop an aesthetic canon.

Works Cited

Bouleau, C. *The Painter's Secret Geometry*. New York: Harcourt, Brace & World, 1963. Print.

D'Arcais, F. F. *Giotto*. New York: Abbeville Press Publishers, 1995. Print.

Dunlap, R. *The Golden Ratio and Fibonacci Numbers.* Singapore: World Scientific Publishing, 1997. Print.

Heath, T. *The Thirteenth Books of Euclid's Elements*. New York: Dover Publications, 1956. Print.

Hertz-Fischler, R. “An Examination of Claims Concerning Seurat and the Golden Number.” *Gazette des Beaux-Arts*. 125 (1983). Print.

Hockney, D. *Secret Knowledge*. New York: Viking Studio, 2001. Print.

Jowett, B. *The Dialogues of Plato*. Oxford: Oxford University Press, 1953. Print.

Kraut, R. *The Cambridge Companion to Plato*. Cambridge: Cambridge University Press,1992. Print.

Figures Resources

Fig.2:
http://www.google.ro/search?q=platonic+solids&espv=2&biw=1366&bih=667&tbm=isch&tbo=u&source=univ&sa=X&ved=OCDIQ7AlqFQoTCKfi8O-Z5cgCFcn_cgodY

Fig.3:
http://www.google.ro/research?q=fibonacci+numbers&espv=2&biw=1366&bih=667&tbm=isch&tbo=u&source=univ&sa-X&ved=OCDIQ7AlqFQoTCNibubOQ5cgCFUjh